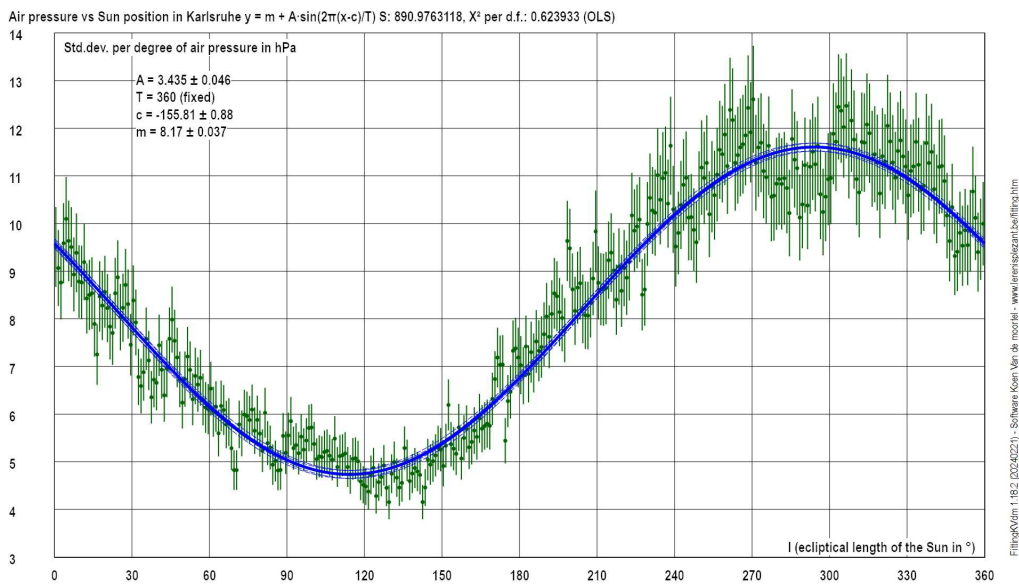


# Measuring and modeling by example



How mathematical functions can be used  
(and misused) to describe the world

Koen Van de moortel  
M.Sc. experimental physics



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[info@lerenisplezant.be](mailto:info@lerenisplezant.be)

[www.researchgate.net/profile/Koen-Van-De-Moortel](http://www.researchgate.net/profile/Koen-Van-De-Moortel)

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Software used:

- The regression graphs and the analyses were made with FittingKVdm, written by the author, see: [www.lerenisplezant.be/fitting.htm](http://www.lerenisplezant.be/fitting.htm).
- Other graphs were made with GeoGebra Classic 5, see: [www.geogebra.org](http://www.geogebra.org).
- The electrical circuits were drawn with [www.circuitlab.com](http://www.circuitlab.com).



## Foreword

I am a difficult person. I don't believe people just because they are "an authority", especially not if they have commercial interests in their preachings, or if their thinking is troubled by dogmatic ideologies, or if they hide behind excuses like "that's the way we've always done things here.". That has not made my life easier, since many people "in charge" have oversized egos and don't like to be questioned, but so be it...

I like modest and creative people who are brave enough to follow their intuition to seek the truth and bring quality rather than superficial fame, like Isaac Newton, Albert Einstein, Alfred Wegener (the continental drift guy), John Harrison (inventor of the first stable clocks that could work on a ship), Shuji Nakamura (Noble Prize 2014 for inventing blue LEDs), my uncle Juul Waumans (not a professor, but a man of practice, who used his common sense to develop one of the best sports floors in the world), etc.

Intuition is often misunderstood as "just a feeling", but feeling has nothing to do with it. Intuition, as I understand it, is "composted experience", the result of many years of observing, tinkering and reflecting, processed in the mysterious depths of our subconscious mind. It makes us see connections and patterns in a blink, as seeds to be nourished by the logical part of our brain.

From the age of 6 I knew for sure I wanted to be a scientist, to explore how and why things work. The "why" is a question I have abandoned as it is probably impossible to answer, but the "how" can be described by mathematical models, the formulas connected with patterns we can observe, and those have been a common thread in the biggest part of my almost 62 years on this planet, so I think I built up some intuition on this subject. I see many people struggling with this process of analyzing their data, while they give their trust more and more to "artificial intelligence" and sometimes forget to use their own intelligence, so that's why I decided to write this book, to share my experience.

Although I can appreciate the beauty of a theory absolutely, I studied experimental physics because I'm more an observer than a theorist, and I'll try to bring my message as intuitively as possible, hoping it will help you in a practical way! And of course I don't want you to believe me just because I say so! Just test it out!

I want to dedicate this book to Roger De Weerd, Rik Verhulst and Rudi Luyten, the high school teachers from the Pius X institute in Antwerpen, who fanned my mathematical and scientific fire and learned me to observe carefully and find order in chaos. And of course I thank my partner-in-life for more than 25 years by now, Dragana, for her patience with me (well... at least most of the time).

Koen Van de moortel, Gent, Belgium, 22 Feb. 2024

PS: Worth watching:

Veritasium documentary about Nakamura: [www.youtube.com/watch?v=AF8d72mA41M](https://www.youtube.com/watch?v=AF8d72mA41M)

and the movie "Longitude", the story of Harrison: [www.youtube.com/watch?v=LHvt48S9l4w](https://www.youtube.com/watch?v=LHvt48S9l4w)

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# Introduction

“Why would we do it?” is always a good question to ask. **Why would we want to see mathematical patterns in our observations?**

First of all, detecting patterns is a skill that is crucial to survive. Animals that don't recognize a predator from a visual, auditive or smell pattern are doomed to get extinct. Not being able to distinguish food from poison will do the same.

As a human, you can go further. Detecting mathematical patterns will help you in a more and more complex environment. It helps to predict the tides and to navigate. It allows you to foresee the best times to seed and to harvest or to hunt. You have an **evolutionary advantage** if you can read the signs that predict a coming storm or a dangerous epidemic.

A few centuries ago it might have seemed a waste of time to figure out whether the attraction between two masses or two electrical charges was inversely proportional to the distance or the square or the third power of that distance, as Newton and Coulomb did, but now we realize we would not have had any of the technology we are so addicted to, without knowing those patterns (or “natural laws” if you will).

Finding the mathematical formula that describes your observation, is a step in the **exploration** of the world. It might lead you to a **hypothesis about how things work**. Finding that the pressure in a gas was proportional to the temperature, brought us to the idea that there might be invisible particles in the air that bump to the walls and store energy by vibrating, even if nobody could see those particles! Finding the formula describing the magnetic field caused by a change in electrical current, led to a lot of technical inventions that made long-distance communications possible.

Once you know the mathematical pattern, the “model”, you can start doing **predictions**. If they turn out to be correct, over and over, they confirm your hypothesis. If not, they force you to rethink the explanation you had in mind. Newton's formulas to describe gravitation were very precise until we started experimenting with very high speeds and masses. If Einstein and his colleagues hadn't improved the model, you wouldn't have this fantastic navigation tool in your smartphone now.

You can also start doing **optimizations**: once you know *how* the composition of a substance influences its qualities, or if you know how things influence the yield of a crop, you can make it better.

Finding relationships that connect different quantities, can also just **make practical measurements easier or less expensive**: measuring temperature, humidity, salinity, wind speed, light intensity, concentration, distance, and so much more, in the 21st century, can all be reduced to measuring an electrical current. But first you have to develop the sensor and the mathematical formula to **calibrate** it!

**Can models prove causal relationships?** Yes and no... If you can control variable  $x$  and you see, an instant or some time later, a change in  $y$  as predicted by the model, you can be quite sure that  $x$  is the cause and  $y$  the consequence. In physics, chemistry, biology etc. such experiments can often be done, but in other sciences one has to use the available data and be happy if you find just correlations. As you probably know, a correlation by itself does not prove causality: the hands of the clock are very well correlated with the changes of day and night, but they don't cause them. Among women, higher education is significantly correlated with a higher risk for breast cancer, although the first doesn't

cause the second (I hope), but this observation confirms a known causal mechanism, namely: higher educated women tend to have fewer children and at a later age, and that makes them more vulnerable. So, studying correlations is useful to make the pieces of the puzzle of causality fit.

The book has four parts:

1) If you are not very familiar with the wide spectrum of those formulas, no problem. I will take you on a journey in **the world of mathematical functions** first, well... at least some relatively simple and interesting kinds, and I'll discuss their useful features.

2) Then I will tell you some stories about **the art of measuring**. How to handle the uncertainties that are present in most data collections? And how to avoid or minimize them as much as possible? How to stick a number to observations that are not so easy to quantify, like "happiness", or "wine quality"? Some tricks and plain common sense can help you here!

3) Next, I'll try to clarify **the process of finding the "best" possible function to use as a model for your observations**. I will also focus on some issues that are often neglected in this process, even by seasoned scholars, and introduce my improvements to existing techniques which I implemented in a software program "FittingKVdm". Questions like "How trustworthy is my model?" need to be asked certainly.

4) The last part of this book consists of **many examples from the real world**, from physics to psychology, from biology to linguistics, from sports to economy. In some of them I analyze easy to repeat experiments that can be done at home or in a classroom, while others use data from public sources.

I hope you enjoy the ride, and I'm certainly looking forward to hearing your comments and suggestions for improvement.